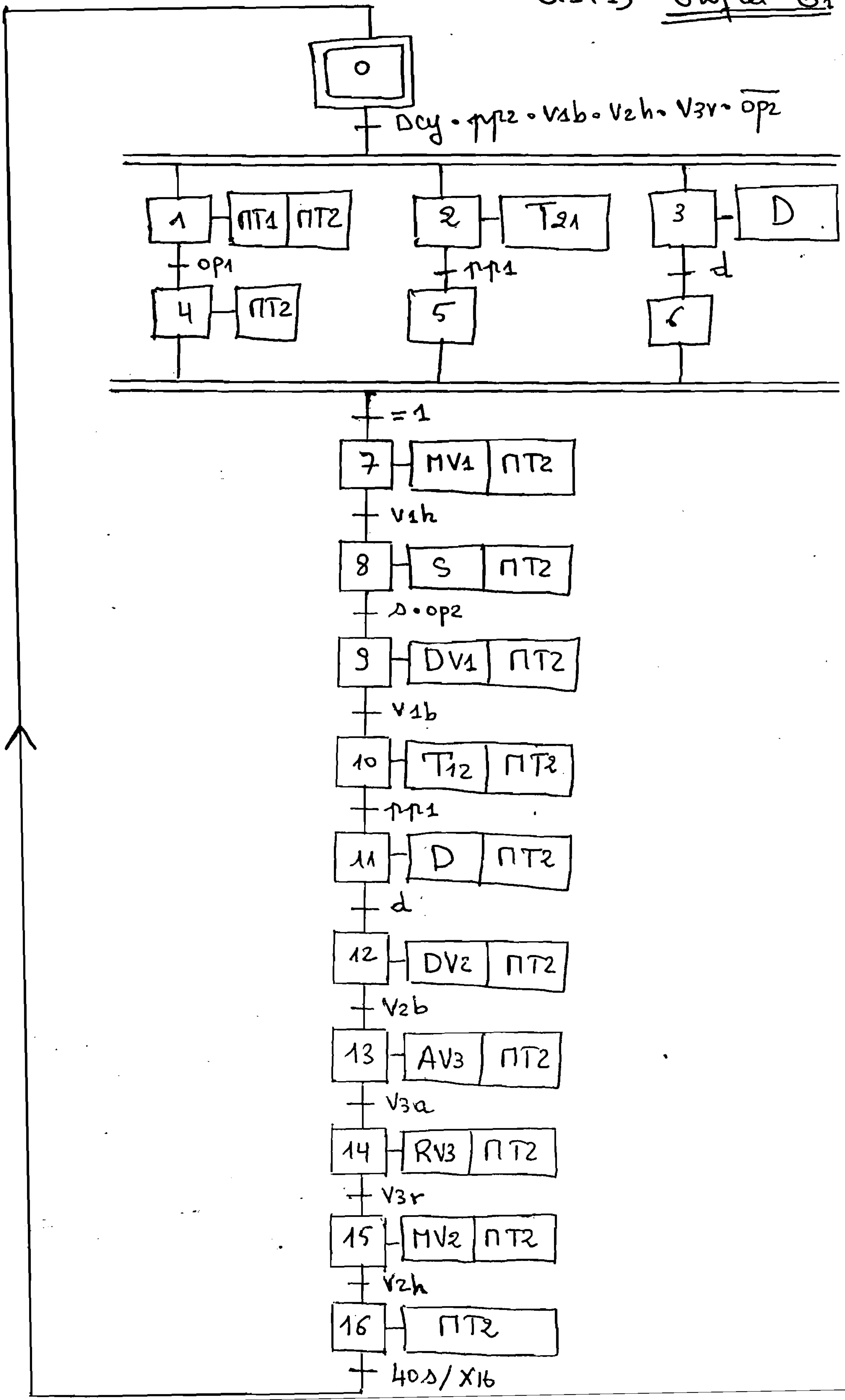
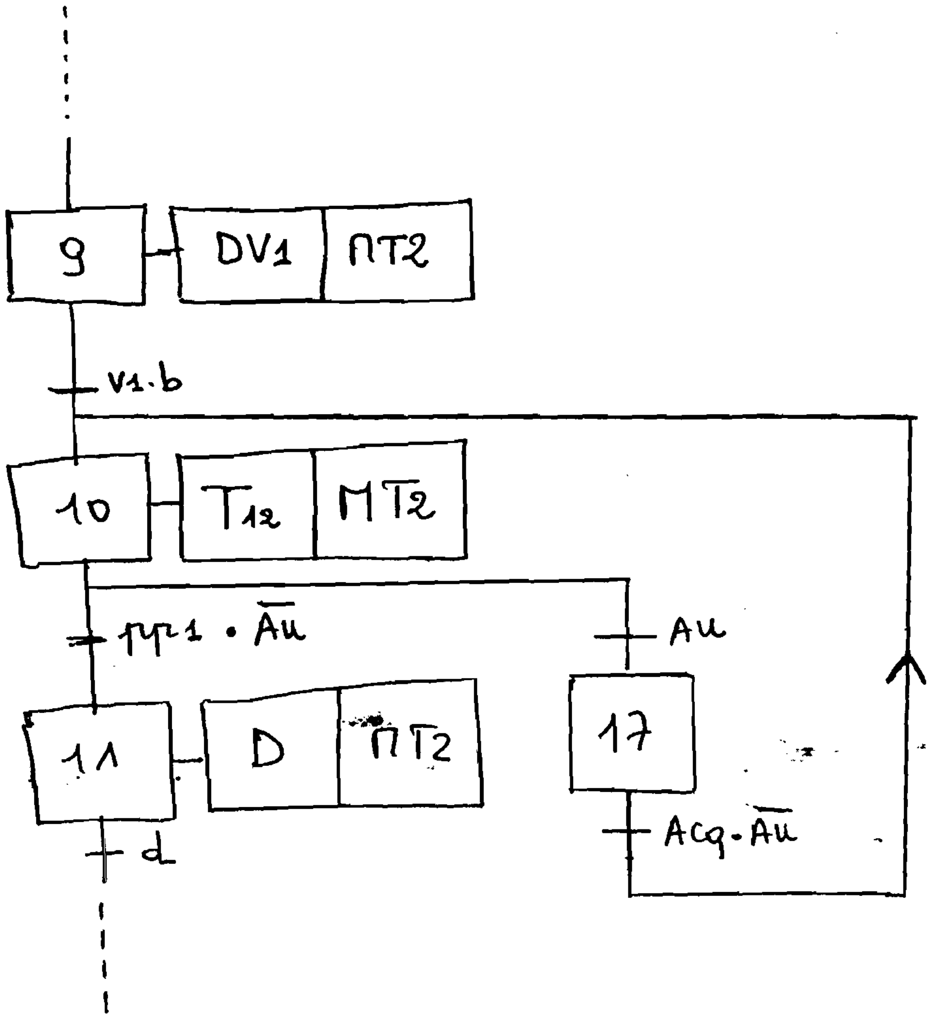
C.I.1) <u>Grafcet G1</u>



C.I.2)

Grafcet G1 modifié



Partie C-II- Asservissement de position d'un axe du bras robotisé

II-1 : Simplification du schéma fonctionnel de la figure 6 :

$$\Omega_m(p) = \frac{1}{J_m p + f_m}\left[\frac{k_c}{R}\left(U(p) - k_e\,\Omega_m(p)\right) - \frac{1}{e}\left(C_r(p) + \frac{1}{e}\left(J_c p + f_c\right)\Omega_m(p)\right)\right]$$

$\Leftrightarrow$

$$\left[1 + \frac{k_e k_c/R}{J_m p + f_m} + \frac{1}{e^2}\,\frac{J_c p + f_c}{J_m p + f_m}\right]\Omega_m(p) = \frac{k_c/R}{J_m p + f_m}\,U(p) - \frac{1/e}{J_m p + f_m}\,C_r(p)$$

$\Leftrightarrow$

$$\left[\left(J_m + \frac{J_c}{e^2}\right)p + \left(f_m + \frac{f_c}{e^2}\right) + \frac{k_e k_c}{R}\right]\Omega_m(p) = \frac{k_c}{R}\,U(p) - \frac{1}{e}\,C_r(p)$$

$\Rightarrow$

$$\Omega_m(p) = T_1(p)\,U(p) - T_2(p)\,C_r(p)$$

avec $\quad T_1(p) = \dfrac{K_1}{1 + \tau_{em}\,p} \quad$ et $\quad T_2(p) = \dfrac{K_2}{1 + \tau_{em}\,p}$

$\tau_{em} = \dfrac{R\,J_e}{k_e k_c + f_e R}$: constante de temps électromécanique

$K_1 = \dfrac{k_c}{k_e k_c + R f_e}$: Gain statique de $T_1(p)$

$K_2 = \dfrac{R}{e\,(k_e k_c + R f_e)}$: Gain statique de $T_2(p)$

$J_e = J_m + \dfrac{J_c}{e^2}$: Inertie équivalent ramené sur l'arbe du moteur

$f_e = f_m + \dfrac{f_c}{e^2}$: Coef. de frottement visqueux équivalent ramené sur l'arbre du moteur.

II-2 : A partir de la réponse indicielle de $T_1(p)$ à un échelon de tension d'amplitude 25V :

On a :

$$\omega_m(\infty) = 25\,K_1 = 200 \ rad.s^{-1} \Rightarrow \boxed{K_1 = 8 \ rad.s^{-1}\,V^{-1}}$$

. à 63% de $\omega_m(\infty)$ on trouve $\boxed{\tau_{em} = 10 \ ms}$

II-3 : schéma fonctionnel de la figure 8.

a/ Fonction de transfert en boucle fermée :

$$\theta_c(p) = H_1(p)\,\theta_{ref}(p) - H_2(p)\,C_r(p)$$

avec $$H_1(p) = \left.\frac{\theta_c(p)}{\theta_{ref}(p)}\right|_{C_r=0} = \frac{\frac{\alpha A K_1}{e\,\tau_{em}}}{p^2 + \frac{1}{\tau_{em}}\,p + \frac{\alpha A K_1}{e\,\tau_{em}}}$$

$$H_2(p) = \left.\frac{\theta_c(p)}{C_r(p)}\right|_{\theta_{ref}=0} = \frac{\frac{K_2}{e\,\tau_{em}}}{p^2 + \frac{1}{\tau_{em}}\,p + \frac{\alpha A K_1}{e\,\tau_{em}}}$$

Equation caractéristique :

$$p^2 + \frac{1}{\tau_{em}}\,p + \frac{\alpha A K_1}{e\,\tau_{em}} = p^2 + 2m\omega_0\,p + \omega_0^2$$

Par identification, on déduit :

$$\omega_0 = \sqrt{\frac{\alpha A K_1}{e\,\tau_{em}}}$$: pulsation propre non amortie (rad/s

$$m = \frac{1}{2}\sqrt{\frac{e}{\alpha A K_1 \cdot \tau_{em}}}$$: coefficient d'amortissement.

b- Calcul de A pour avoir $m = 0,7$.

$\alpha = 0,8\ V/rad$; $K_1 = 8\ rad.s^{-1}V^{-1}$; $\tau_{em} = 10\ ms$ et $\rho = 50$

$$A = \frac{\rho}{4m^2 \alpha K_1 \tau_{em}}$$

A.N : $$A = \frac{50}{4.(0,7)^2.\ 0,8 . 8 . 0,01} = 398,6$$

$$\omega_0 = \sqrt{\frac{0,8 . 398,6 . 8}{50 . 0,01}} = 71,43\ rad/s$$

Dépassement : $$D\% = 100 . e^{-\frac{m\pi}{\sqrt{1-m^2}}}$$

A.N : $$\boxed{D\% = 4,6\ \%}$$

Temps de pic : $$T_p = \frac{\pi}{\omega_0\sqrt{1-m^2}}$$

A.N : $$\boxed{T_p = 0,062\ s}$$

c- Etude de la précision statique

$$\boxed{\varepsilon_1(p) = \theta_{ref}(p) - \theta_c(p) = [1 - H_1(p)]\,\theta_{ref}(p) + H_2(p)\,C_r(p)}$$

avec $$H_1(p) = \frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2}$$

$$H_2(p) = \frac{K_2}{\alpha A K_1} \cdot \frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2} = \frac{K_2}{\alpha A K_1} H_1(p)$$

$$\boxed{\varepsilon_1(p) = \frac{p(p + 2m\omega_0)}{p^2 + 2m\omega_0 p + \omega_0^2}\,\theta_{ref}(p) + \frac{K_2}{\alpha A K_1}\,\frac{\omega_0^2}{p^2 + 2m\omega_0 p + \omega_0^2}\,C_r(p)}$$

$$\theta_{ref}(t) = t \cdot u(t) \xrightarrow{T.L.} \theta_{ref}(p) = \frac{1}{p^2}$$

$$C_r(t) = 100 \cdot u(t) \xrightarrow{T.L} C_r(p) = \frac{100}{p}$$

Théorème de la valeur finale

$$\varepsilon_1(\infty) = \lim_{p \to 0} p\, \varepsilon_1(p)$$

soit
$$\boxed{\varepsilon_1(\infty) = \frac{2m}{\omega_0} + \frac{100\, k_2}{\alpha A K_1}}$$

A.N : $\varepsilon_1(\infty) = 0{,}0196 + 0{,}026 = 0{,}049$

$$\boxed{\varepsilon_1(\infty) = 0{,}049 \quad \text{soit } 4{,}9\%}$$

Partie A : Géométrie des masses.

I. 1. Position du centre G de l'ensemble (4)

- cylindre (masse M)

$$\vec{KG_c} = \frac{H}{2}\vec{x_3} + L\,\vec{Y_1}$$

- tige (masse m_t)

$$\vec{KG_t} = \left(\frac{L}{2} - a_4\right)\vec{Y_1}$$

- disque (masse m)

$$\vec{KG_d} = -a_4\,\vec{Y_1}$$

On applique la relation du barycentre

$$\vec{KG} = \frac{M\,\vec{KG_c} + m_t\,\vec{KG_t} + m\,\vec{KG_d}}{M + m_t + m}$$

$$x = \frac{M\,\frac{H}{2}}{M+m_t+m}$$

$$y = \frac{ML + m_t\left(\frac{L}{2} - a_4\right) - m\,a_4}{M + m_t + m}$$

I. 2. Matrices centrales d'inertie

- cylindre :

$$[II(C,\vec{y})] = \begin{bmatrix} \frac{Mr^2}{2} & 0 & 0 \\ 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} & 0 \\ 0 & 0 & \frac{Mr^2}{4} + \frac{MH^2}{12} \end{bmatrix}_{(\vec{x}, \vec{y}, \vec{z})}$$

$$A_4 = \frac{Mr^2}{2} + ML^2 + \frac{m_t L^2}{12} + m_t\left(\frac{L}{2} - a_4\right)^2 + \frac{mr^2}{4} + m\,a_4^2$$

$$B_4 = \frac{Mr^2}{4} + \frac{MH^2}{12} + \frac{mr^2}{2} + \frac{MH^2}{4}$$

$$C_4 = \frac{Mr^2}{4} + \frac{MH^2}{12} + \frac{MH^2}{4} + ML^2 + \frac{m_t L^2}{12} + \left(\frac{L}{2} - a_4\right)^2 m_t + \frac{mr^2}{4} + m\,a_4^2$$

$$F_4 = M\frac{H}{2}.L$$

Partie A.II. Cinématique.

II.1 Vitesses

- $\vec{V}_{A/R_0} = \vec{V}_{O/R_0} + \vec{\Omega}_{1/R_0} \wedge \vec{OA}$.

$\vec{V}_{O/R_0} = \vec{0}$, $\vec{\Omega}_{1/R_0} = \dot{\beta}\,\vec{x_0}$, $\vec{OA} = a_1\vec{Y_1}$.

$$\vec{V}_{A/R_0} = a_1\,\dot{\beta}\,\vec{Z_1}$$

- $\vec{V}_{I/R_0} = \vec{V}_{O/R_0} + \vec{\Omega}_{1/R_0} \wedge \vec{OI}$

$\vec{OI} = a_3\vec{X_0} + b_1\vec{Y_1}$.

$$\vec{V}_{I/R_0} = b_1\,\dot{\beta}\,\vec{Z_1}$$

$$[\mathbb{I}(Cyl)] = \begin{bmatrix} & & \\ 0 & \frac{Mr^2}{4}+\frac{MH^2}{12} & 0 \\ 0 & 0 & \frac{Mr^2}{4}+\frac{MH^2}{12} \end{bmatrix}_{(\vec{x}_3,\vec{Y}_1,\vec{Z}_3)}$$

- tige.

$$[\mathbb{I}(tig)] = \begin{bmatrix} \frac{m_t L^2}{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{m_t L^2}{12} \end{bmatrix}_{(\vec{x}_3,\vec{Y}_1,\vec{Z}_3)}$$

- disque.

$$[\mathbb{I}(disque)] = \begin{bmatrix} \frac{mr^2}{4} & 0 & 0 \\ 0 & \frac{mr^2}{2} & 0 \\ 0 & 0 & \frac{mr^2}{4} \end{bmatrix}_{(\vec{x}_3,\vec{Y}_1,\vec{Z}_3)}$$

I.3 Matrice dinertiè de l'ensemble (4).

leplan $(K,\vec{x}_3,\vec{Y}_1)$ est un plan de symétrie, par suite $(K,\vec{Z}_3)$ est un axe principal d'inertie

$$[\mathbb{I}_K(4)] = \begin{bmatrix} A_4 & -F_4 & 0 \\ -F_4 & B_4 & 0 \\ 0 & 0 & C_4 \end{bmatrix}_{(\vec{x}_3,\vec{Y}_1,\vec{Z}_3)}$$

avec:

$$\vec{V}_{I/R_0} = b_1\dot{\beta}\vec{Z}_1$$

- $\vec{V}_{K/R_0} = \vec{V}_{O/R_0} + \vec{\Omega}_{1/R_0} \wedge \vec{OK}$

$$\vec{OK} = a_3\vec{X}_0 + b_2\vec{Y}_1$$

$$\vec{V}_{K/R_0} = b_2\dot{\beta}\vec{Z}_1$$

- $\vec{V}_{E\in 2/R_1} = \vec{V}_{A/R_1} + \vec{\Omega}_{2/R_1} \wedge \vec{AE}$

$$\vec{V}_{A/R_1} = \vec{0}, \quad \vec{\Omega}_{2/R_1} = \dot{\theta}\vec{X}_0, \quad \vec{AE} = r_1\vec{X}_0 + r_2\vec{Y}_1$$

$$\vec{V}_{E\in 2/R_1} = r_2\dot{\theta}\vec{Z}_1$$

- $\vec{V}_{E\in 3/R_1} = \vec{V}_{I/R_1} + \vec{\Omega}_{3/R_0} \wedge \vec{IE}$

$$\vec{V}_{I/R_1} = \vec{0}, \quad \vec{\Omega}_{3/R_0} = \dot{\varphi}\vec{Y}_1, \quad \vec{IE} = -r_3\vec{X}_0 - r_2\vec{Y}_1$$

$$\vec{V}_{E\in 3/R_1} = r_3\dot{\varphi}\vec{Z}_1$$

- $\vec{V}(D\in 5/R_0) = \vec{V}_{A/R_0} + \vec{\Omega}_{2/R_0} \wedge \vec{AD}$

$$\vec{V}_{A/R_0} = a_1\dot{\beta}\vec{Z}_1, \quad \vec{AD} = a_2\vec{X}_0 - r_5\vec{Y}_1, \quad \vec{\Omega}_{2/R_0} = (\dot{\beta}+\dot{\theta}$$

$\vec{V}_{D\in 3/R_0} = a_1 \dot\beta \vec{Z}_1 - r_5 (\dot\beta + \dot\theta) \vec{Z}_1$ (Page 2)

II. 2 Roulement sans glissement aux points D et E

- en E

$$\vec{V}_{E\in 2/R_0} = \vec{V}_{E\in 3/R_0}$$

$$r_2 \dot\theta = r_3 \dot\psi \Rightarrow$$

$$\dot\psi = \frac{r_2}{r_3} \cdot \dot\theta = \frac{r_2}{r_3} \omega$$

- En D. $\vec{V}_{D\in 6/R_0} = \vec{0}$

$$a_1 \dot\beta - r_5 (\dot\beta + \dot\theta) = 0$$

$$(a_1 - r_5) \dot\beta = r_5 \dot\theta$$

$$\dot\beta = \frac{r_5}{a_1 - r_5} \cdot \dot\theta = \frac{r_5 \, \omega}{a_1 - r_5}$$

I. 3 Vitesse de G.

$$\vec{V}_{G/R_0} = \vec{V}_{K/R_0} + \vec{\Omega}_{3/R_0} \wedge \vec{KG}$$

$$\vec{V}_{K/R_0} = b_1 \dot\beta \vec{Z}_1 \quad , \vec{\Omega}_{3/R_0} = \dot\psi \vec{Y}_1 + \dot\beta \vec{X}_0$$

$$\vec{KG} = x \vec{X}_3 + y \vec{Y}_1$$

$$\vec{V}_{..} = b_1 \dot\beta \vec{Z}_1 + (\dot\psi \vec{Y} + \dot\beta \vec{X}_0) \wedge (x \vec{X}_3 + y \vec{Y}_1)$$

(II)

$$\Gamma_x = -x \dot\psi^2 \cos\psi$$

$$\Gamma_y = x \ddot\beta \dot\psi \cos\psi - \dot\beta \left((b_4 + y) \dot\beta - x \dot\psi \cos\psi \right)$$

$$\Gamma_z = x \dot\psi^2 \sin\psi + x \dot\beta^2 \sin\psi$$

Partie A. III. Dynamique

$$\psi = 0, \quad \dot\psi \neq 0$$

$$\vec{Z}_3 = \vec{Z}_1, \quad \vec{Y}_3 = \vec{Y}_1, \quad \vec{X}_3 = \vec{X}_1$$

III. 1 Action sur (4)

- Poids appliqué en G. $\vec{P} = -\underbrace{(M + m_4 + m)}_{m_4} g \vec{Z}_0$

$$\vec{\mathcal{M}}_G(\vec{P}) = \vec{0} \quad , \vec{P} = -m_4 g (\cos\beta \vec{Z}_1 + \sin\beta \vec{Y}_1)$$

$$\vec{\mathcal{M}}_K(\vec{P}) = (x \vec{X}_3 + y \vec{Y}_1) \wedge (m_4 g$$

$$= -m_4 g \cdot \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ \sin\beta \\ \cos\beta \end{pmatrix}$$

$$= -m_4 g \begin{pmatrix} y \cos\beta \\ -x \cos\beta \\ x \sin\beta \end{pmatrix}$$

$$\vec{KG} = x\vec{X}_3 + y\vec{Y}_1$$

$$\vec{V}_{G/R_0} = b_2\dot\beta\vec{Z}_1 + (\dot\psi\vec{Y}_1 + \dot\beta\vec{X}_0)\wedge(x\vec{X}_3 + y\vec{Y}_1)$$

$$\vec{V}_{G/R_0} = b_2\dot\beta\vec{Z}_1 - x\dot\psi\vec{Z}_3 + x\dot\beta\sin\psi\,\vec{Y}_1 + y\dot\beta\vec{Z}_1$$

$$\vec{V}_{G/R_0} = (b_2+y)\dot\beta\vec{Z}_1 - x\dot\psi\vec{Z}_3 + x\dot\beta\sin\psi\,\vec{Y}_1$$

$$\vec{Z}_3 = \cos\psi\,\vec{Z}_1 + \sin\psi\,\vec{X}_0$$

$$\vec{V}_{G/R_0} = -x\dot\psi\sin\psi\,\vec{X}_0 + x\dot\beta\sin\psi\,\vec{Y}_1 + \left((b_2+y)\dot\beta - x\dot\psi\cos\psi\right)\vec{Z}_1$$

$$V_x = -x\dot\psi\sin\psi, \quad V_y = x\dot\beta\sin\psi$$

$$V_z = (b_1+y)\dot\beta - x\dot\psi\cos\psi.$$

I.4. Accélération de G.

$$\vec{\Gamma}_{G/R_0} = \frac{d\vec{V}_{G/R_0}}{dt}\Big/_{R_0}$$

$$\vec{\Gamma}_{G/R_0} = \dot V_x\vec{X}_0 + \dot V_y\vec{Y}_1 + \dot V_z\vec{Z}_1 + \dot\beta V_y\vec{Z}_1 - V_z\dot\beta\vec{Y}_1$$

avec $\dot V_x = -x\dot\psi^2\cos\psi$, $\dot V_y = x\dot\beta\dot\psi\cos\psi$,
$\dot V_z = x\dot\psi^2\sin\psi.$

$$= -m_4 g\begin{vmatrix} -x\cos\beta \\ x\sin\beta \end{vmatrix}$$

$$\{\mathcal{T}(P)\}_K = \begin{Bmatrix} 0 & -m_4 g\,y\cos\beta \\ -m_4 g\sin\beta & m_4 g\,x\cos\beta \\ -m_4 g\cos\beta & -m_4 g\,x\sin\beta \end{Bmatrix}_{(\vec{X}_0,\vec{Y}_1,\vec{Z}_1)}$$

- Action de 3 sur (4) en K

$$\{\mathcal{T}_{4/3}\}_K = \begin{Bmatrix} X_{14} & L_{14} \\ Y_{14} & 0 \\ Z_{14} & N_{14} \end{Bmatrix}_{(\vec{X}_0,\vec{Y}_1,\vec{Z}_1)}$$

- action de (3) sur (4) en K.

$$\{\mathcal{T}_{4/3}\}_K = \begin{Bmatrix} 0 & 0 \\ N_J & C_J \\ 0 & 0 \end{Bmatrix}_{(\vec{X}_0,\vec{Y}_1,\vec{Z}_1)}$$

III.3 Théorème de la résultante dynamique

$$\begin{cases} X_{14} = -m_4 x\dot\psi^2 \\ N_J + Y_{14} - m_4 g\sin\beta = \left(2x\dot\beta\dot\psi - \dot\beta^2(b_1+y)\right)m_4 \\ Z_{14} - m_4 g\cos\beta = 0 \end{cases}$$

III.3) Moment dynamique en K.

$$\vec{\sigma}_K(4/R_0) = m_4\, \vec{KG} \wedge \vec{V}_{K/R_0} + \left[I_K(4)\right] \vec{\Omega}_{4/R_0}$$

$$\bullet\ \vec{\delta}_K(4/R_0) = \frac{d\vec{\sigma}_K(4/R_0)}{dt} + m_4\, \vec{V}_{K/R_0} \wedge \vec{V}_{G/R_0}$$

$$\vec{KG} \wedge \vec{V}_{K/R_0} = (x\vec{x}_3 + y\vec{y}_h) \wedge (b_2 \dot{\beta}\, \vec{z}_1)$$

$$= -b_2\dot{\beta} x \cos\psi\, \vec{Y}_1 + b_2 \dot{\beta} y\, \vec{X}_0$$

$$\vec{\Omega}_{4/R_0} = \dot{\psi}\vec{Y}_1 + \dot{\theta}(\cos\psi\, \vec{x}_3 + \sin\psi\, \vec{z}_3)$$

$$\left[I_K(4)\right] \cdot \vec{\Omega}_{4/R_0} = \begin{bmatrix} A_4 & -F_4 & 0 \\ -F_4 & B_4 & 0 \\ 0 & 0 & C_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}\cos\psi \\ \dot{\psi} \\ \dot{\theta}\sin\psi \end{bmatrix}$$

$$= \begin{bmatrix} A_4\dot{\theta}\cos\psi - F_4\dot{\psi} \\ -F_4\dot{\theta}\cos\psi + B_4\dot{\psi} \\ C_4\dot{\theta}\sin\psi \end{bmatrix}$$

$$\vec{\delta}_K(4/R_0) = \left(B_4\dot{\beta}\dot{\psi} - F_4\dot{\theta}\dot{\beta} - m_4 b_2 \dot{\beta}^2 x + (C_4 - A_4)\dot{\theta}\dot{\psi} + F_4\dot{\psi}^2 \right) \vec{Z}_1$$

Equations dynamiques

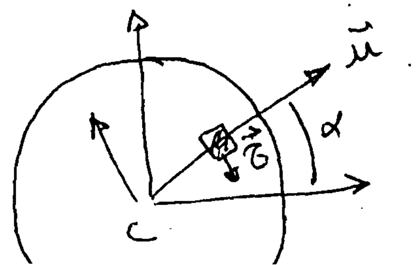
$$\begin{cases} A_4 - m_4 g y \cos\beta = 0 \\ C_J + m_4 g x \cos\beta = 0 \\ N_{14} = \left(B_4\dot{\beta}\dot{\psi} + F_4(\dot{\psi}^2 - \dot{\theta}\dot{\beta}) - m_4 b_2 x \dot{\beta}^2 + (C_4 - A_4)\dot{\theta}\dot{\psi} \right) \end{cases}$$

III.4 Expression de N_J

On considère une surface d'appui circulaire de rayon R.

la pression étant uniforme:

$$p = \frac{N_J}{S} = \frac{N_J}{\pi R^2}$$



$$\vec{\sigma}_4(4/R_0) = -m_4 b_2 \dot\beta x \cos\psi \vec{Y}_1 + m_4 b_2 \dot\beta y \vec{X}_0 + (A_4 \dot\theta \cos\psi - F_4 \dot\psi)\vec{x}_3 + (-F_4 \dot\theta \cos\psi + B_4 \dot\psi)\vec{Y}_1 + C_4 \dot\theta \sin\psi \vec{Z}_3$$

$$\frac{d\vec{\sigma}_K}{dt}\Big|_R = (m_4 b_2 \dot\beta \dot\psi \cancel{x \sin\psi})\vec{y}_1 + \dot\beta(B_4\dot\psi - F_4\dot\theta\cos\psi - m_4 b_2 \dot\beta x \cos\psi)\vec{Z}_1 - A_4 \dot\theta\dot\psi \cancel{\sin\psi}\,\vec{x}_3 + (A_4\dot\theta\cos\psi - F_4\dot\psi)(\dot\theta \cancel{\sin\psi \vec{Y}_1} - \dot\psi \vec{Z}_3) + C_4 \dot\theta\dot\psi\cos\psi \vec{Z}_3 + C_4\dot\theta \cancel{\sin\psi}(\dot\psi\vec{X}_3 - \dot\theta\cos\psi \vec{Y}_1)$$

$$\frac{d\vec{\sigma}_K}{dt}\Big|_{R_0} = \dot\beta(B_4\dot\psi - F_4\dot\theta - m_4 b_2\dot\beta x)\vec{Z}_1 + -(A_4\dot\theta - F_4\dot\psi)\dot\psi\vec{Z}_1 + C_4\dot\theta\dot\psi\vec{Z}_1$$

$$m_4 \vec{V}_{K/R_0} \wedge \vec{V}_{G/R_0} = m_4 \cdot b_2 \dot\beta \vec{Z}_1 \wedge ((b_2+y)\dot\beta - x\dot\psi)\vec{Z}_1 = \vec{0}$$

$$\vec{\tau} = -\tau\vec{v} = -f.p.\vec{v}$$

à la limite de l'adhérence $\frac{|\tau|}{p} = f.$

$$d\vec{F} = -\tau.ds\,\vec{v} = -\tau\, r\,dr\,d\alpha.\vec{v}$$

$$d\vec{M}_c = \vec{CM} \wedge d\vec{F} = -r\vec{u} \wedge \tau\, r\,dr\,d\alpha\,\vec{v} = -\tau r^2 dr\,d\alpha.\vec{z}$$

$$\vec{M}_J = -\int \tau r^2 dr\,d\alpha = \frac{2\pi\tau.R^3}{3}$$

$$C_J = -2\cancel{\pi}\frac{f N_J}{\cancel{\pi}R^2}\cdot\frac{R^3}{3} = -\frac{2}{3}f N_J R.$$

$$N_J = |C_J|\,\frac{3}{2fR}.$$

III.5 inconnus de liaisons.

- $N_J = 3\dfrac{m_4 g x\,|\cos\beta|}{2fR}.$
- $L_{14} = m_4 g y \cos\beta$

$$Y_{14} = m_4 g \sin\beta + m_4(2x\dot\beta\dot\psi - \dot\beta^2(b_4+y)) - \frac{3m_4 g x 1}{2fR}$$

$$Z_{14} = m_{g} \cos\beta$$